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Dimensional Analysis for the Design of Satellites in LEO

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1 Thesis Guideline

In this work the powerful technique of dimensional analysis is used to benefit from the similarity of spacecrafts and missions to facilitate and accelerate the design of newly developed satellites. By means of the Buckingham-Π-Theorem, a widely employed method in dimensional analysis, ratios and non-dimensional similarity parameters have been identified in this work which formalize and facilitate the comparison between the characteristics of satellites of different sizes. As a consequence new satellite designs can be based on results from former missions and existing designs can be verified. Furthermore it is shown in the work that dimensional analysis can be used in a newly developed approach to quantify the mission performance of a satellite and therefore provide a method for the verification of satellite standards by relating physical characteristics of the spacecraft to the achievable mission performance of the satellite.
A special emphasis during the work is put on the application of the theoretical results on the CubeSat standard as this satellite class is gaining more and more significance. However, the mission efficiency of the CubeSat standard was not yet sufficiently questioned. Consequently, this work analyzes the efficiency of the CubeSat standard with the help of the Mission Performance Index and gives guidelines to a possible new CubeSat standard which combines all the advantages of the current CubeSats as their cost-, size- and mass-effectiveness but also a higher mission performance. The work finishes with a brief presentation of a CubeSat specific commercial off-the-shelf component database, which was also created during this work.

2 Theoretical Basis

2.1 CubeSat-Specific Definitions and Standards

The CubeSat standard originates from the efforts of the California Polytechnic State University, San Luis Obispo (Cal Poly) and the Space Systems Development Laboratory (SSDL) at Stanford University in 1999 within the CubeSat program in order to develop a new class of satellites [Heidt et al., 2001], [Toorian et al., 2005].
In its standard configuration the satellite measures 100.00 mm x 100.00 mm x 113.50 mm with a weight of up to 1.33 kg and notably uses commercial off-the-shelf (COTS) electrical components. This configuration is known as 1 unit CubeSat or 1U CubeSat. Other built and launched standardized configurations are 2U (100.00 mm x 100.00 mm x 227.00 mm, mass up to 2.66 kg) and 3U (100.00 mm x 100.00 mm x 340.50 mm, mass up to 4.00 kg) CubeSats. Smaller formats like 0.5U CubeSats and larger ones (4U, 5U and 6U) are also possible. The CubeSat Design Specification (CDS) [CalPoly, 2009] defines all these standards by stating the nominal dimensions of the standard 1U CubeSat, dimension tolerances, the reference coordinate system, acceptable materials and other information.

2.2 Buckingham-Π-Theorem

The Buckingham-Π-Theorem is a fundamental theorem in dimensional analysis named after the American physicist Edgar Buckingham. It states that an equation written in the general form

$$f(Q_1, Q_2, \ldots, Q_n) = 0 \tag{2.1}$$

where the n symbols $Q_1, \ldots, Q_n$ denote physical quantities of n different kinds with k independent dimensions can be rewritten as

$$F(\Pi_1, \Pi_2, \ldots, \Pi_i) = 0 \tag{2.2}$$

where the Πs represent $i = n - k$ independent dimensionless products of the form

$$\Pi_i = Q_1^{a_1} Q_2^{a_2} \ldots Q_n^{a_n} \tag{2.3}$$

The use of Π as a dimensionless product was introduced by Edgar Buckingham in his original paper on the subject in 1914 [Buckingham, 1914] from which the theorem draws its name.

2.3 Advantages and Disadvantages of the Use of Dimensional Analysis during the Design Phase of Satellites

Dimensional Analysis is a possibility to formalize the potential hidden in the similarity of spacecrafts and missions in order to accelerate and facilitate the design phase of a satellite. With this powerful technique it is possible to quickly and easily design a new spacecraft based on former designs, verify already existing designs and quantify the mission performance of a satellite in comparisons to other designs. As knowledge from former successful missions is used for all these three applications of dimensional analysis, the robustness of the newly developed or verified design is believed to increase. Furthermore, this satellite design approach needs very few input parameters and is believed to be quicker than classical approaches.

However, the validation of the ratios and non-dimensional parameters needs extensive databases of flown satellites for reliable results. Furthermore, the non-dimensional parameters and the Mission Performance Index are only a model of reality and thus a simplification of all the interdependencies which determine the characteristics of a satellite and mission in the real world.

3 Dimensional Analysis

3.1 Design Approaches: Top-Down and Bottom-Up

In general, two different problem assignments are possible for the design of a satellite, distinguishing them by means of the nature of the input and output parameters of the satellite design: in the payload-centric Top-Down approach the parameters of the mission are given (i.e. the type of the mission, the payload to be used on the spacecraft) and the ideal spacecraft fulfilling the requirements of the mission has to be found; in the spacecraft-centric Bottom-Up approach the parameters of a possible spacecraft are fixed (i.e. the size of the spacecraft, the power which can be provided) as well as the orbital parameters and a mission fitting to the spacecraft and orbit has to be determined.

Most of the usual industrial and scientific missions are done in a Top-Down design approach whereas CubeSats are more often build in a Bottom-Up approach. Both approaches are investigated in the thesis in order to keep the results as general as possible.

3.2 Proceeding of the Validation of the Theoretical Results

For the validation, thus the determination of a fixed numerical value for the parameters for a given satellite class, data of flown satellites is necessary. In this work some of the theoretical results are validated by use of data from non-geosynchronous (NGSO) satellites [Springmann and de Weck, 2004]. Data of flown CubeSats was tried to be gathered together but was not extensive enough to be used for the validation of the theoretical results. The data collected so far, however, is presented in the work and used with limitations for the application of the results.
In order to quantify the quality of the results of the application of the non-dimensional parameters on the NGSO-data, different statistical parameters are used. In a first step, the arithmetical average of the results over all satellites is built. Secondly, the standard deviation is calculated and finally the coefficient of variation $\frac{s}{x_{av}}$ is determined.

3.3 Dimensional Analysis of Single Satellites

3.3.1 Ratios: Mass, Volume and Power

Inspired by the often presented approach in literature [Kiesling, 197172], [Pritchard, 1984], [Larson and Wertz, 1999], [Saleh et al., 2002] to estimate the mass of the subsystems of a satellite as percentage of the dry mass $m_{S/C_{dry}}$ or wet mass $m_{S/C_{wet}}$ of the spacecraft, the usefulness of ratios for the design of spacecrafts is firstly investigated. Mass ratios are validated for the Bottom-Up as well as the Top-Down approach with NGSO-data, leading to satisfying results. Power ratios are investigated, too, showing also promising results. Due to interdependencies between mass and power presented in literature [Larson and Wertz, 1999, p.334],[Springmann and de Weck, 2004], it was assumed that the combination of power and mass ratios would lead to more satisfying, thus less dispersed results than the mass and the power ratios alone. This assumption is not confirmed for the NGSO-data. Further validations are therefore recommended in order to decide if the single ratios or the combinations are best to describe the relations between mass and power for a given satellite database.
A special volume ratio, namely the packing factor, is defined in the work as

$$p := \frac{V_{used}}{V_{S/C}} \tag{3.1}$$

or more detailed

$$p = \frac{V_{Bus}}{V_{S/C}} + \frac{V_{P/L}}{V_{S/C}} \tag{3.2}$$

Unfortunately, this parameter could not be validated because of the lack of NGSO-data, but it is believed to be worth further investigations.

3.3.2 Non-Dimensional Parameters

During the analysis of ratios, interdependencies between mass, power and the volume of the satellite became obvious. The orbital period interplaying in the satellite system, too, it is assumed that a non-dimensional parameter build with these four quantities can be very useful for design purposes. Dimensional analysis according to the Buckingham-Π-Theorem directs us to the following non-dimensional product

$$\Pi_1 = \frac{P_{S/C} \cdot t^3{}_{Orbit}}{V_{S/C}{}^{\frac{2}{3}} \cdot m_{S/C}} \tag{3.3}$$

With the definition of an equivalent edge length x_{eq} defined as

$$x_{eq} := V_{S/C}{}^{\frac{1}{3}} \tag{3.4}$$

in order to facilitate the investigations, equation (3.4) can be re-expressed as

$$\Pi_2 = \frac{P_{S/C} \cdot t^3{}_{Orbit}}{x_{eq}^2 \cdot m_{S/C}} \tag{3.5}$$

Analyses are also made with dimensional parameters derived from the non-dimensional parameters Π_1 and Π_2 by reducing the number of influencing quantities in the similarity parameters (i.e the orbital period, mass density). These dimensional parameters are useful for the design of satellites with similar quantities but also for further analyses of the characteristics of the satellites in the databases.

Especially the two parameters Π_1 and Π_2 are assumed to be very useful for quick design estimations as they combine four main quantities of a satellite design: mass, power, length or volume and orbital period. Three parameters of them, mass, power and length or volume, are input quantities for the design process for the Top-Down as well as the Bottom-Up approach, for the Top-Down approach with payload specific parameters and for the Bottom-Up approach with spacecraft specific quantities. Π_1 and Π_2 are not directly integrated in the flow of the design calculations based on dimensional analysis. However, they are especially advantageous when one of the four design quantities is missing as input parameter. It can then easily be calculated with the knowledge of the numerical value of the non-dimensional parameter and three given quantities.

In addition to the Top-Level approach, non-dimensional parameters are presented in this work for the subsystems Power, represented by a dimensional anaylsis for the batteries and the solar arrays, Communication as well as AOCS, represented by the analysis for a reaction wheel. For the payload, three different missions are considered, namely Earth Observation, Space Science and Technology Demonstration. For the first two, additional non-dimensional parameters are developed with similar quantities. For the Technology Demonstration the payload can be any kind of component from any kind of subsystem, thus no further non-dimensional parameter is required to represent it. It is perfectly described by the ratios and previously created parameters for the subsystems and the spacecraft.

3.4 Mission Performance Index

In a newly developed approach to quantify the mission performance of a satellite, physical characteristics of the spacecraft are related to the achievable mission performance of the satellite by creating the so called Mission Performance Index (MPI). The MPI is defined for both the Bottom-Up and the Top-Down approach as

$$\begin{aligned} MPI = (\Psi_{P/L})^{\alpha} \cdot (\Psi_{AOCS})^{\beta} \cdot (\Psi_{Power})^{\chi} \cdot (\Psi_{Structure})^{\delta} \cdot \\ \cdot (\Psi_{Thermal})^{\epsilon} \cdot (\Psi_{C\&DH})^{\phi} \cdot (\Psi_{Com})^{\varphi} \cdot (\Psi_{Prop})^{\gamma} \end{aligned} \tag{3.6}$$

The Mission Performance Index is designed in a Top-Level approach by taking ideally all stakeholders into account. In a first order approach, the factors Ψ building the MPI are based on the non-dimensional parameters for the subsystems and the payload. The exponents $\alpha, \beta, \chi, \ldots$ range between 0 and 1 and express the functional importance of the subsystems for the mission accomplishment.
The higher the MPI, the "better performant" is the mission. Therefore Mission Performance

Parameters (MPP) are defined for every type of mission. They represent the quantities which shall be maximized for a successful mission. If a subsystem is not present in a satellite, its factor Ψ is set equal to a very small value, for example 10^{-20}, assuming that a complete lack in a subsystem is not conducive for the mission performance. Finally, a normalization within a satellite class is recommended as this limits the value range of the MPIs and thus enables the comparison of the mission performance of different satellites more easily
When enough data on flown satellites is available, an entire database of MPIs can be created. In this context, it is assumed that it will be possible to assign MPI ranges to satellite classes. The database can then be used to verify if a given design achieves the intended mission performance by comparing the calculated MPI of the design with the satellite class specific MPI ranges. In addition, the database can be helpful to determine the required satellite class for an intended mission performance.

3.5 Application of the Results

In the thesis it is also shown how the ratios, non-dimensional parameters and the Mission Performance Index can be practically used. In order to determine the numerical values for the ratios and non-dimensional parameters for a satellite class, thus to validate them, a hypothetical 1U CubeSat is used for the Bottom-Up approach and a hypothetical NGSO-satellite is applied for the Top-Down approach. Once the non-dimensional parameters are determined for a satellite class, they can be used for designing a spacecraft with as little as four main input parameters and some further secondary design inputs. This is done in a next step for a hypothetical 2U CubeSat, based on the results obtained from the calculations with the 1U CubeSat one step earlier. A netflow diagram showing the inderdependencies of the quantities in the system can be helpful here (see Figure 3.1 for the Bottom-Up approach used in an Earth Observation Mission). Finally, the calculation of the MPI and its normalization within the CubeSat-class prove, that the 1U CubeSat is less performant than the larger 2U CubeSat. Combined with the fact that CubeSats provide higher structure and communication mass ratios for smaller standards, this result can be very helpful for the definition of a larger, more performant CubeSat-standard.

3.6 Dimensional Analysis of Clusters

As this work was intended to be embedded in the current F6-program of the Defense Advanced Research Projects Agency (DARPA) in collaboration with Orbital Sciences Corporation, the work gives also an introduction to the analysis of satellite clusters with the Buckingham-Π-Theorem similar to the investigation of the single satellites.

A cluster has to be analyzed in two steps, firstly taking into consideration only the cluster as a complete system and secondly considering each satellite individually. For the investigation of the individual satellites of the cluster the non-dimensional parameters and ratios for single satellites can be used. To characterize the cluster as a whole, the non-dimensional parameters and ratios of a single satellite are re-expressed with quantities describing the whole cluster. These quantities can simply be the sums of the corresponding quantities of the single satellites, for example, the total mass of the cluster $m_{Cluster}$. The two different design approaches, the payload-centric Top-Down and the spacecraft-centric Bottom-Up approach are both valid for the design of the clusters.

The Mission Performance Index of a cluster architecture can be calculated in two steps. Firstly $MPI_{Cluster}$ is calculated which implies the consideration of the cluster as a whole. It is normalized to become a figure between 0 and 100. In a second step also the performances of the single satellites generating the cluster are taken into account. For this purpose, $MPI_{Single\ Sat}$ for every single satellite is calculated. Afterwards, an arithmetical average is build over all the satellites which results in the quantity $MPI_{all\ Sats}$. A weighing can be used here in case some satellites are considered to be more critical for the mission accomplishment than others. $MPI_{all\ Sats}$ is then normalized to a number between 0 and 1 and then multiplied with $MPI_{Cluster}$ to create $MPI_{Sat+Cluster}$, again a number between 0 and 100. As now the importance of the different subsystems for the mission success changes in comparison to the ones for a monolithic satellite, the numerical values for the exponents $\alpha, \beta, \chi, \ldots$ for $MPI_{Cluster}$ change, too.
The current form of the Mission Performance Index only implies physical quantities which directly influence the design of a spacecraft such as the mass of the spacecraft or the orbital altitude. A further development, however, by integrating all stakeholders of a mission and thus also parameters such as costs, robustness and launcher availability into the MPI is recommended. The current systems engineering approach is then changed to a more sophisticated engineering systems approach.

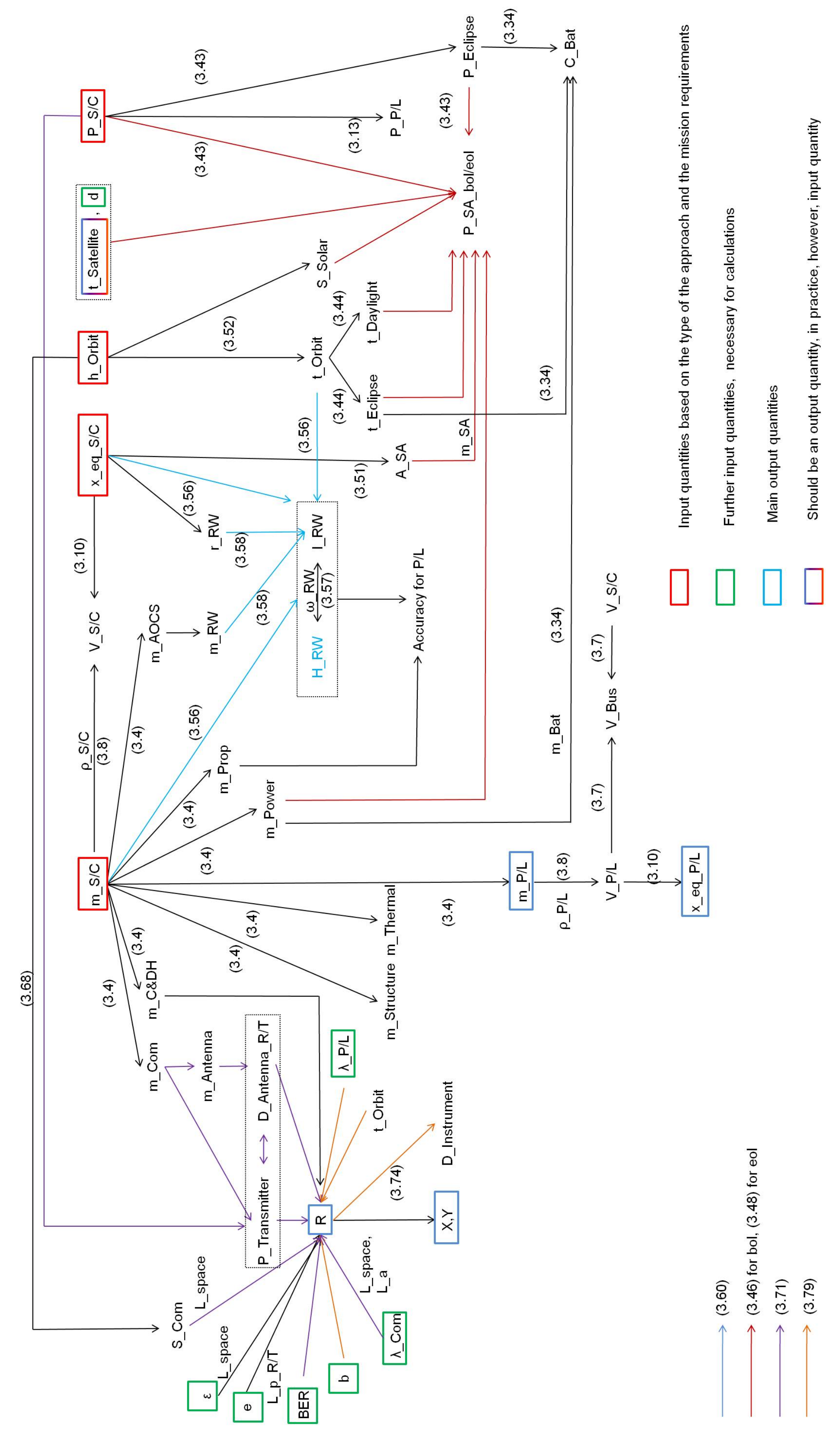

Figure 3.1: Bottom-Up network flow diagram for an Earth Observation mission

4 Future Prospects

Future work based on this thesis might include the further improvement and development of the non-dimensional parameters for every subsystem as well as the validation of them by means of extensive databases for several satellite classes. Beside the implementation of further stakeholders into the MPI, the development of an "universal MPI" which combines all mission types as well as the two design approaches Top-Down and Bottom-Up, is also desirable.
Furthermore, as clusters gain increasing importance in the space sector, the detailed application of the theoretical results of the dimensional analysis on cluster architectures is also supposed to be worth the investigation.
Finally, the combination of the component database with the theoretical results into a design tool for CubeSats is also desirable. The design tool will ideally enable the quick design of a spacecraft based on the theoretical results of this work and indicate the COTS-components which are best suited for the design. A continuously updated database is therefore necessary.

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